

AGRICULTURAL PRODUCTION FUNCTIONS

by

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I. SUMMARY

In this paper I shall discuss the concept of a production function, its relation to the theory of supply and its use as a tool for economic analysis. In the second section I shall review some of the measurement problems, describe in the agricultural context, one method that has been evolved to overcome these problems and present some preliminary results of an attempt at measurement using this method. Mathematical notation has been kept to a minimum.

II. THEORETICAL REMARKS

It was 200 years ago that Captain Cook was sailing round these islands. At the same time a French nobleman, Turgot (1), was laying down the Law of Variable Proportions. He pointed out that after a period of increasing returns, successive applications of labour to land yielded diminishing increments to product. Nearly fifty years later, in 1815, Malthus (2) and Sir Edward West (3)

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made similar statements about the application of a combination of capital and labour. The principle was taken up by Ricardo soon after in his Principles of Political Economy.

These last three were concerned with applications to the Theory of Distribution. Von Thünen's (4) contribution in the 1840's has remained the definitive statement of the Law. If either capital or labour is increased while the other remains constant the product is subject to diminishing returns. Von Thünen then related wages and interest to the value of the last increment of product.

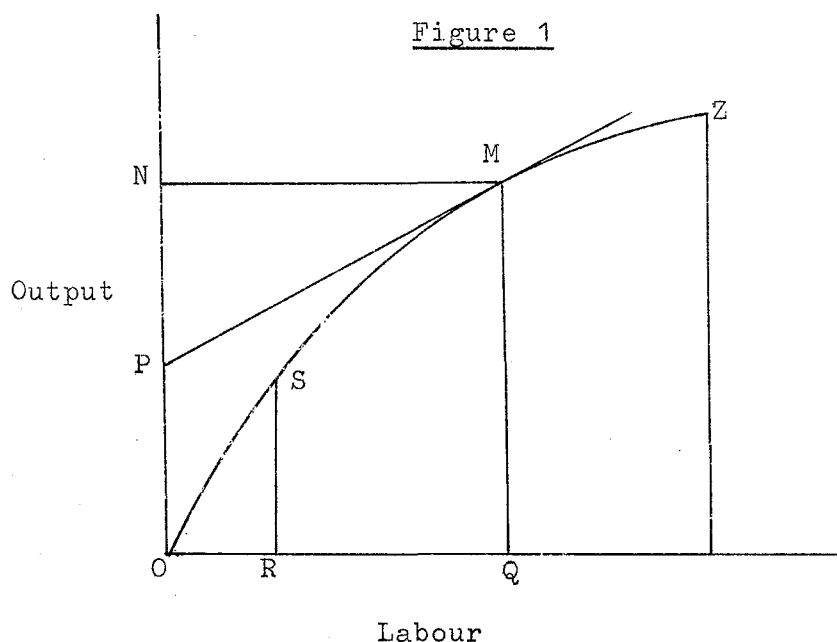
The Theory of Production has been refined since and terms like isoclines, isoquants, ridge lines, production possibility frontiers and elasticity of substitution, form part of an impressive armoury of jargon for the production economist. But still any beginning student in economics who can draw an S shaped growth curve, label his axes correctly and call it a production function is well on the way to passing his first exam. With information about prices of factors and product and some assumptions about the behaviour of entrepreneurs he has at his fingertips all the elements of the Theory of Supply. Like the famous parrot he is half an economist. The difficulties (or even the need) of measurement of the production function and the difficulties of relating it to the economy and the price mechanism as a whole have so far eluded him.

III. THE CONCEPT OF A PRODUCTION FUNCTION AND ITS RELATION TO THE THEORY OF SUPPLY

In this paper, we first consider the production function that describes the production response at all levels and combinations of inputs in relation to the theory of supply, and then consider variation in individual inputs. The former refers to returns to scale and the latter to returns to factors.

3.

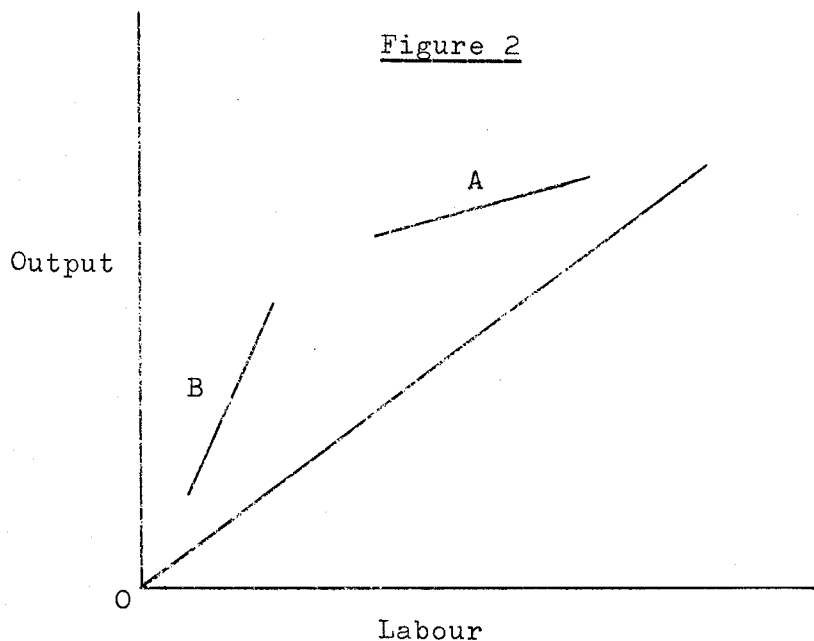
Suppose we take the general case as described by A.A. Walters (5) and others. This is shown in Figure 1 and consists of a simple productive activity that requires only one homogeneous input (say labour) to produce one homogeneous output. The shape of the function indicates decreasing returns to scale.



The line OMZ is supposed to represent the maximum attainable output from any given level of input. It is known as the production function. This representation has some curious properties; it implies that at any input level like R, output can take any value along RS. There are other unsatisfactory properties but it will do for the present. The line PM is the rate of exchange of labour in terms of output. According to the marginal theory it will pay to increase output to the point M on the production function. That is, it will pay to increase the input of labour till the value of the last increment to product just equals the cost of a (last) unit of labour. The fact that the line PM is straight implies that the levels of input and output have no effect on their price. This is one of the conditions for perfect

competition. The Theory of Supply depends on this condition; it also depends on the assumption that the individual entrepreneur engaged in this productive activity will operate at M. If he does so he will be making profit OP; since the cost of labour (in terms of output) is PN and the value of output is ON. This situation cannot last under conditions of perfect competition. The existence of excess profit will attract firms into industry. The price of labour will rise, the price of the product will fall. The price line PM will become steeper. The point of maximum profits will tend towards O. We say that maximum profit is inconsistent with decreasing returns to scale and economic equilibrium under perfect competition.

If the production function shows constant returns to scale, that is, if all inputs are increased by some proportion output will increase by the same proportion, then it may be described by a straight line emanating from the origin as in Figure 2.



With a price line like A maximum profits will be achieved at an infinite level of output, profits will be infinite and firms will enter until the price looks like B, in which case the point

of maximum profits will tend towards point O. The price line will then return to A. We say that the level of maximum profits is indeterminate. For the case of increasing returns to scale it is not possible to achieve maximum profits at all.

The theory of supply is in a parlous state. It does not auger well for the use of a production function in supply analysis. The situation can be retrieved to some extent by appealing to Ricardo and his contemporaries. Suppose agricultural land is absolutely limited in supply. Then the excess profit spills over into Ricardian rent. There is a unique point of maximum profit, supply once more becomes determinate. It should be recorded at this stage that it does not pay to enquire too deeply into the meaning of the phrase "absolutely limited in supply".

This line of reasoning is necessary to proceed, but leaves us with further difficulties as far as the examination of production data is concerned. However we choose to build the theory of supply onto the foundation of the production function; it is not too difficult to accept the proposition that there exists some function which takes account of every conceivable productive influence and perfectly describes the production response at different levels of these influences. The aim will be to describe the function as closely as possible.

IV. USE OF THE PRODUCTION FUNCTION

In general, there will be three tasks that economic models of any description will be designed to assist with.

1. Prediction
2. Policy Recommendations
3. Projection

Prediction

The problems of prediction and forecasting in economics

are well documented. If output some years ahead is to be predicted, the predictive model must include the relationships for all the variables that effect output. The relationships that should be included in the model must be selected on the basis of an adequate theory - but the theory outlined above is not too sound. In the system just described there is a technological relationship, the production function, and some economic relationships. The economic relationships can be divided into the decision functions, and the factor supply and product demand relationships. The decision functions measure the stimuli which influence the entrepreneurs to select any particular input levels to achieve any particular level of output. These stimuli might be prices. As with any system of equations the production function is at best only a part of the model required for prediction.

Policy Recommendations

Recommendations at the firm level could include the adjustment of input use to achieve maximum profit or some other goal. At the national level they will include resource adjustment on the basis of partial equilibrium analysis like the prospectus for the National Development Conference.

The method to be employed here requires a basic assumption as to profit maximisation. If entrepreneurs are all profit maximisers our regression techniques break down. We must resort to other techniques such as Klein's factor share method.

Secondly, if entrepreneurs are not seen to be perfect maximisers do we attribute this to different decision functions, different production functions or merely errors of measurement? If these problems cannot be resolved, then policy recommendations cannot be made.

We must be careful not to estimate resource returns on the basis of the absence of perfect adjustment and then discover

that no adjustment is required. Alternatively, we might falsely measure imperfect adjustment, and make policy recommendations, when perfect adjustment was in fact being achieved.

There is one case which may justify policy recommendations at the national level. It may be that farmers are not maximising profit because they are hindered from doing so by entrepreneurial or institutional restrictions. These may take the form of credit restrictions or some restriction on acquiring land or some other resource.

In this instance the method proposed does not fall down. The parameters estimated are reliable estimates of the true parameters. There may be a case for advising removal of the restrictions.

It is doubtful if the derived production function has adjustment implications for the individual farmer. In theory the method to be proposed measures individual functions for each farm. In fact, the management factor which we attempt to measure is the most important determinant of profit. No managerial recommendations can spring from the function itself.

Projection

It is in the field of projection that the agricultural production function is most useful, though as far as I am aware it is yet to be used for this purpose. As long as the relationships that are confusing the estimates remain stable, and this can be one of the conditions of the projection, then the function will perform well as a projection model. In other words if there is bias in the estimates due to some unspecified relationships but that relationship continues to hold for the period of projection then this achieves the same result as a model that has unbiased estimates but includes the relationship that would cause the bias.

There is, of course, a problem that is always present in projection work, the estimated relationships may not hold

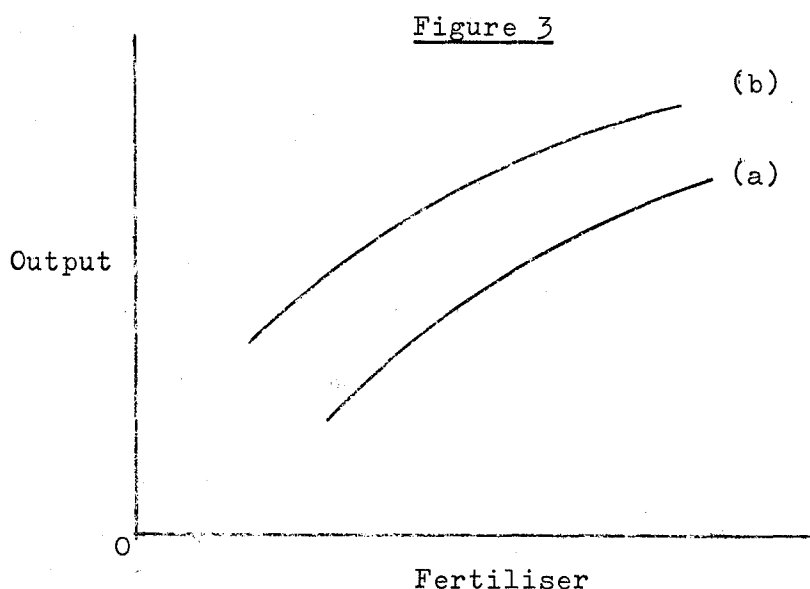
outside the range of the data used for estimation.

V. MEASUREMENT OF THE PRODUCTION FUNCTION

Professor Douglas (6) and his co-worker Charles Cobb, first used the method of least squares to estimate the relationship between capital, labour and production in American manufacturing in 1928. After this there was a spate of attempts to measure everything for which data was available. At the same time there was a growing body of literature concerned exclusively with criticisms of Professor Douglas's and his followers' work. Some economists even managed to gain credit both as proponents and as critics of the method.

All this culminated in a pioneer article by Jacob Marschak and William Andrews (7) in 1944. Much of the rest of this paper is a non-mathematical exposition of their contribution.

The discussion will be conducted in the framework of Figure 3. To give it an agricultural flavour the factor to be varied is fertiliser.



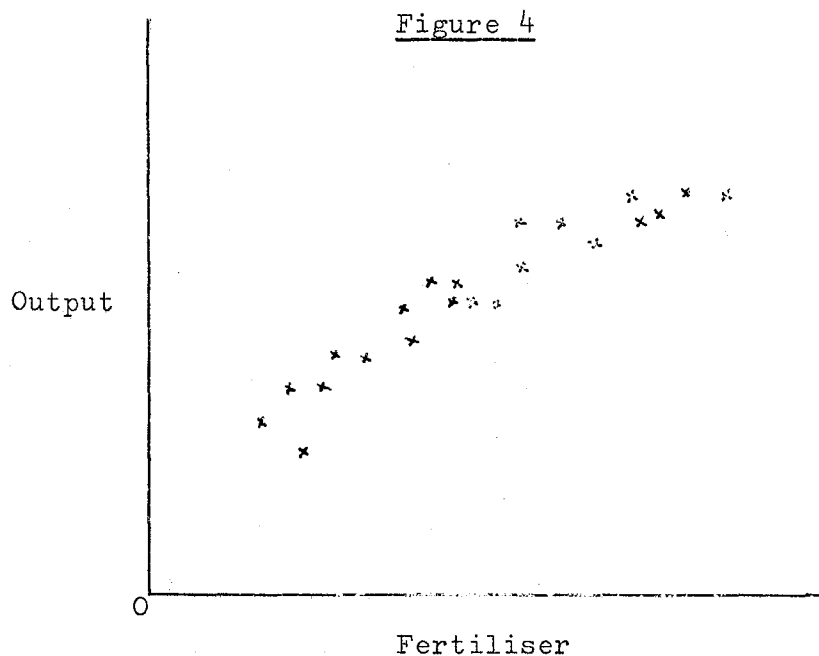
The other factors of production; land, labour and capital, are assumed constant at some level. The line (a) might represent the production response to fertiliser at some given level and combination of land labour and capital. The line (b) might represent the response at some other level and combination. It is easy to generalise the results of the discussion by making all factors variable. For a start it will be assumed that prices of factors and product are independent of individual farmers' actions.

We shall use the particular form of the function known as the Cobb-Douglas function. Most of the following remarks apply to production functions in general. The Cobb-Douglas function allows for diminishing returns. Another of its properties is that at any point on the curve, the slope and position are related to one another so that the ratio slope/position, known as the elasticity of production, is always constant. The function's characteristics offer considerable computational advantages at the cost of some loss as a representation of reality.

Suppose there is a farmer who neither ages nor learns from experience or Lincoln College. He maintains his capital stock over time in exactly the same condition, his flock is stable and retains the same genetic quality, his dogs are no more or less fractious as the years go by. For some reason, perhaps because of haphazardly fluctuating fertiliser prices, he alters his level of fertiliser use. Furthermore, the fertiliser he puts on in one year affects production only in that year. We observe his levels of fertiliser use and observe the corresponding levels of production. If in two years the same amount of fertiliser is used, but the level of production is different, we put the difference down to the weather or some other random influence. Or we might have observed a number of farmers in one year all of whom have practically identical farms with the same level of capital and

other production inputs, who for no apparent reason have different levels of fertiliser use. We note the fertiliser use levels and the corresponding level of production, but though fertiliser use may be the same on two farms, production may be different. At this stage we are careful not to ascribe the difference in production to any particular cause.

In these happy circumstances we could get a scatter diagram like Figure 4.



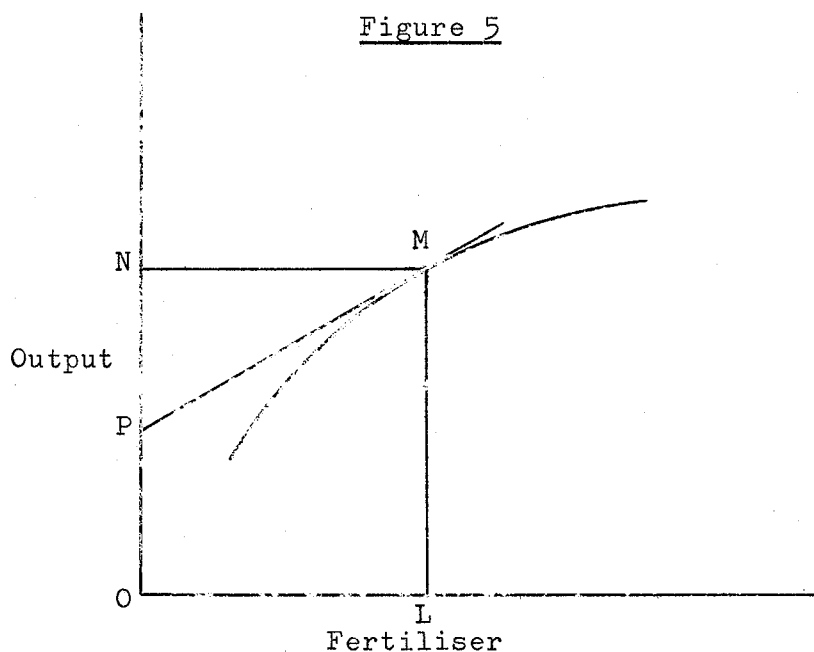
By some means or other we could draw a line through these points and measure its slope. Even if farm size or quantity or quality of labour was not constant there are techniques available for fitting a plane or hyperplane to these observations if they can be measured. We could draw a line or describe a surface which represented the production response to a change in any number of productive influences.

So far two statements are crucial to the estimating procedure. One was concerned with the reason for the variation in fertiliser use, the other with the variation in response to

identical levels of fertiliser use.

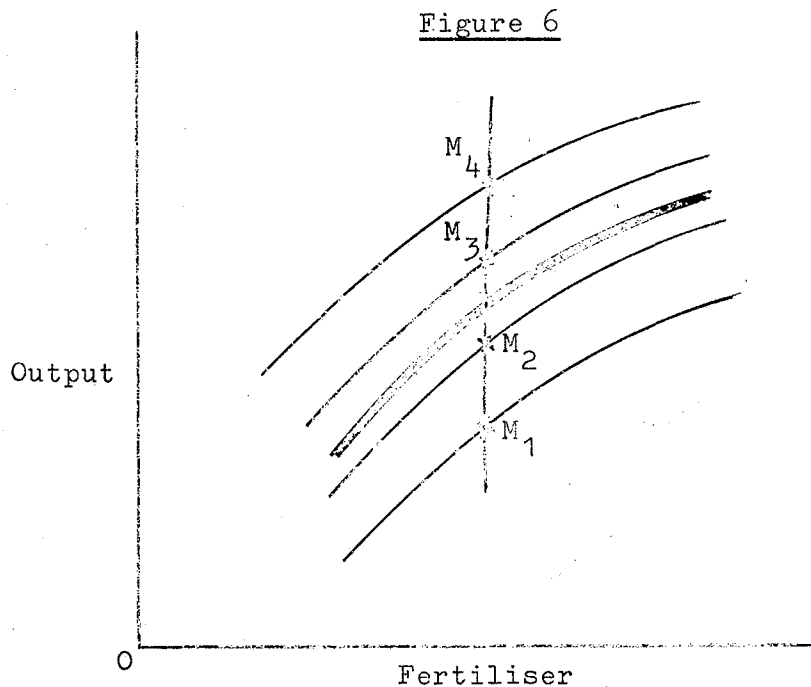
Further discussion will be confined to the cross-section of farms in one year. Some of the arguments apply to time-series data.

Since farms' records are the only kind of data that is likely to be available, the circumstances that induce the farmer to select any particular level of fertiliser use are important. Each farmer in the cross-section faces the same fertiliser price and receives the same price for his product. If, as has been suggested, all farmers have the same production function and know it, and all seek to maximize profit, then they would all use the same amount of fertiliser and achieve the same level of production. We would get the situation depicted in Figure 5.

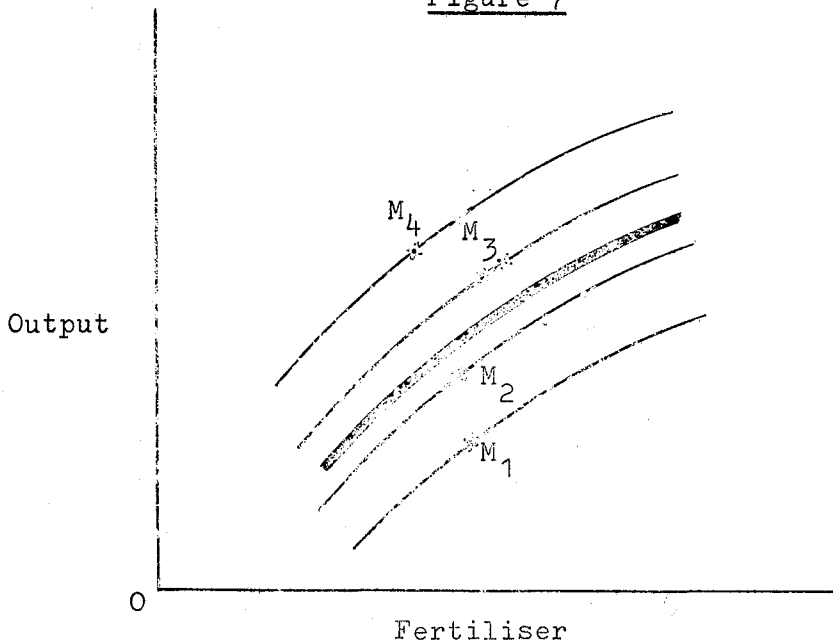


The true, unknown production function is OMZ. The price of fertiliser in terms of product is PM. All farmers would use OL of fertiliser and achieve ON production. This situation is inconsistent with the casual observation that all farmers do not use the same quantity of inputs nor do they produce the same amount of output. We must explain the variation in some other way.

It is hardly realistic that though our sample of farms has been chosen from as homogeneous a group as possible, that all have the same production functions. Differences in soil type, terrain and managerial ability mean that each farm has an individual production function. What we are now trying to describe is the average production function. This situation is depicted in Figure 6.

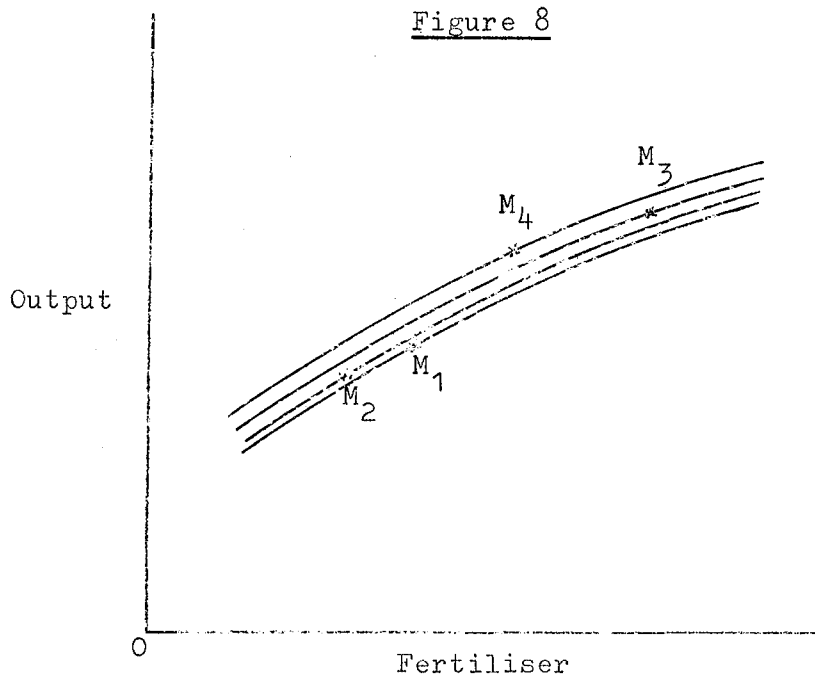


The heavy line is the average production function. The lighter lines are the individual production functions. The line through the points M_1 , M_2 , M_3 and M_4 , which would be the observations secured, in no way represents the average production function. But there are other reasons why the observations show variability. Farmers may not know what their production function looks like. They are likely to make mistakes in choosing the level of inputs that maximises profit even if that is what they are trying to do. In this case we will get a conglomeration of points like that in Figure 7.

Figure 7

Any line drawn through these points could only by coincidence look like the average production function. The direction the line will take will depend on which has the greater variation - the production function or the decision function. As so far described the problem is known technically as that of identification. There are a number of ways of identifying a system or identifying one or more of the equations in a system. Most are difficult to describe except with the use of mathematical symbols but the method most appropriate to the farm production and decision making system is as follows. If we could add more dimensions to the production function by measuring the causes of variation but ignore the decision variation we would have a much narrower band of production functions and the situation would be like Figure 8.

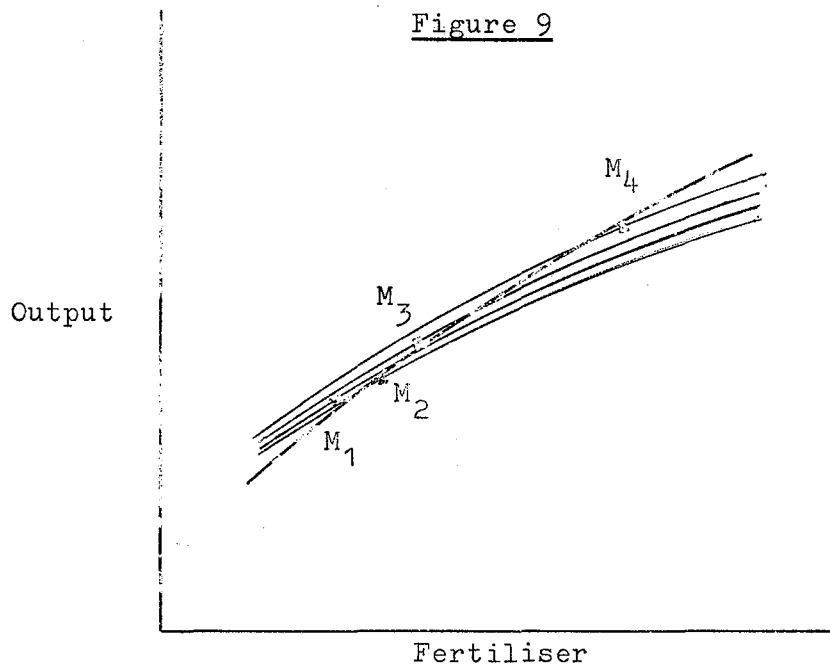
The best line through these points would indeed look like the average production function. For this to be so, one crucial attribute of the decision making process must be present. The decisions of any one farmer must be independent of the position

Figure 8

of his production function, relative to the average production function. But this is unlikely to be the case. One of the important determinants of the position of the production function and one which cannot, as yet, be measured is the managerial ability of the farmer. i.e. the efficiency with which he combines a given bundle of inputs to achieve some output. It is in this aspect that the trouble lies. A better than average manager will, with the same bundle of inputs be able to achieve a higher level of production than the average. He will therefore use more inputs and achieve a higher output than the average. (The price line will be tangent to the higher production functions further to the north-east.)*

* This is a special consequence of the choice of the Cobb-Douglas type of function and statistical distribution of the divergencies from the average function.

The situation is as depicted in Figure 9.



The best line drawn through points M_1 , M_2 , M_3 and M_4 is said to be a biased estimate of the production function because of the association between the level of fertiliser use and the deviation from the average of each individual production function. Accurate measurement of the average production function from cross-section data is always difficult.

Consider now a similar sort of problem for a time series of observations on one farmer. Earlier it was stated that the reason for different levels of fertiliser use in different years might be changing prices of fertiliser and product. Because of this the identification problem is not so acute though it can still be present under certain conditions. But there is a similar sort of problem with regard to bias. Suppose some technological advance has taken place over the period of the observations. Then the situation is analogous to that depicted in Figure 9 (simplified by constant prices) with the higher production functions representing later years and situations of technological advance.

There is a technique which has been used for some time by biological and psychological scientists, and recently by economists Mundlak (8) and Hoch (9), which is well suited to this problem if both time series and cross-section data is available. The technique is known as analysis of covariance. We say that the rate of technological change is more or less the same for all farms - the production function shifts upwards by the same amount in one year for each farm. We also say that the difference between the farm production functions is the same in all years. In other words there are four sources of variation in output:

- (a) variation due to changes in the inputs; fertiliser, land, labour and capital,
- (b) variations due to the shift of the production function over time,
- (c) variations due to systematic differences in the production function between farms, and
- (d) those due to random effects like the weather.

In terms of Figure 9 we measure the distance between the production functions by imagining the lines as shelves stretching back into the paper.

The method still requires independence of variation in the production function and levels of input use. That is, when the farmer makes his decision to use any particular amount of fertiliser he must not be influenced in this decision by any of the productive factors that have not been accounted for in the measured production function. To get estimates in which we have a high degree of confidence the unaccounted variation in the production functions (d) must be small compared with the variations in the decision functions (a). That is, for some reasons (these may be institutional or entrepreneurial restrictions) individual farmers should have been prevented from using the profit maximising level of input.

VI. SOME RESULTS

For those interested the mathematical form of the function is:

$$y = k_o k_f k_t \prod_{q=1}^Q x_q^{a_q}$$

where

- y is output
- k_o is the equation constant
- k_f is the constant that varies between firms
- k_t is a constant that varies between years
- x_q is the level of the q th input
- a_q is the elasticity of production for the q th input
- Q is the total number of input

k , k_f , k_t and a_q , are the parameters to be estimated.

The equation is linear in logarithms and can be estimated by analysis of covariance. The statistical model includes a multiplicative error term which is supposed to be log-normally distributed with a mean unity and a finite variance. It is also supposed to have other properties such as serial independence and independence with the values of the x_q .

Tests can be carried out of the hypothesis that there are no significant differences between firms and years. Tests can also be carried out of the hypothesis that the exponents are not significantly different from zero.

The data is from 30 farms for the twelve year period 1953/54 to 1964/65. The farms are from the Southland Intensive group taken from the records of the New Zealand Meat and Wool Boards' Economic Service. Southland was chosen in the first instance because of the homogeneity of the soil type, terrain and weather in the area.

Two sets of equations were estimated. In the first set capital items were amalgamated into one variable. Output in both sets was the weighted sum of the quantity of lamb, mutton, beef and wool produced.

Variables Used in Equation Sets 1 and 2

<u>Variable</u>	<u>Units of Measurement</u>	
	<u>Set 1</u>	<u>Set 2</u>
Capital	Real value (£'s)	
Improvements		Real value (£'s)
Plant & Machinery		Real value (£'s)
Stock		Stock units
Land	Acres	Acres
Labour	Man Years	Man Years
Current Fertiliser	Tons	Tons
Previous Fertiliser	Tons	Tons

For each set four equations were estimated.

- (i) Ignoring both firm and time effects
- (ii) Ignoring firm effects
- (iii) Ignoring time effects
- (iv) Including both firm and time effects.

The best estimate chosen was with assumption (ii), that the firm constant was unity.

The coefficients for both sets under assumption (ii) are shown in Table 1.

Table 1

	<u>ELASTICITIES OF PRODUCTION</u>	
<u>Variable</u>	<u>Set 1</u>	<u>Set 2</u>
Capital	.59*	
Improvements		.22*
Plant & Machinery		.13*
Stock		.63*
Land	.05	-.17*
Labour	.07	.04
Current Fertiliser	.07*	.05*
Previous Fertiliser	.08*	.05*
Sum	.87	.95
R ²	.76	.81

* significant at the 1% level or better

The R^2 is the proportion of the variation in output explained by the variables.

A sum of the coefficients of less than one indicates decreasing returns to scale.

The coefficients shown in Table 1 are elasticities. They indicate the percentage change in output which will result in a 1% change in the quantity of input, other inputs being held constant.

The time constants are shown in Table 2.

Table 2

<u>Year</u>	<u>TIME CONSTANTS</u>	
	<u>Set 1</u>	<u>Set 2</u>
1953/54	1.00	1.00
1954/55	1.12	1.11
1955/56	1.06	1.03
1956/57	1.05	1.07
1857/58	1.22	1.19
1958/59	1.24	1.20
1959/60	1.22	1.20
1960/61	1.24	1.21
1961/62	1.28	1.23
1962/63	1.31	1.28
1963/64	1.23	1.24
1964/65	1.20	1.23

The time constants measure the effects of the factors that influence output of all farms in any year in the same way. One factor that does just this and has not been measured explicitly is weather. The time constants thus measure the effect of both weather and technological change. It could be expected that the effect of technological change is even while that of weather is uneven. Further work in this field will include an attempt to extract the weather effect with the use of meteorological data.

Taking 1953/54 as the base year and smoothing the constants by fitting a line to them* gives a compound rate of growth due to changing technology of 2.6% for set 1 and 2.3% for set 2. This is considerably higher than that found by other workers (10) who have measured the change as a function of time. It will be interesting to compare these results for those of other districts in New Zealand. Work is in hand for this exercise.

VII. CONCLUSIONS

The technique described promises to provide good estimates of production function parameters and of the rate of technological change. Further refinement of data and technique are necessary before the results can be used with confidence. Projection of output or of input requirements for the future is the field in which the results are most likely to be useful.

* The line fitted is of the form $\log S_n = n \log (1 + r)$ where n goes from zero to eleven, S_n is the constant in year 1953/54 + n and r is the rate of growth.

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